

<<索伯列夫乘子理论>>

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## &lt;&lt;索伯列夫乘子理论&gt;&gt;

## 前言

'I never heard of "Uglification", "Alice ventured to say. 'What is it?'" Lewis Carroll, "Alice in Wonderland"

Subject and motivation. The present book is devoted to a theory of multipliers in spaces of differentiable functions and its applications to analysis, partial differential and integral equations. By a multiplier acting from one function space  $S_1$  into another  $S_2$ , we mean a function which defines a bounded linear mapping of  $S_1$  into  $S_2$  by pointwise multiplication. Thus with any pair of spaces  $S_1, S_2$  we associate a third one, the space of multipliers  $M(S_1, S_2)$  endowed with the norm of the operator of multiplication. In what follows, the role of the spaces  $S_1$  and  $S_2$  is played by Sobolev spaces, Bessel potential spaces, Besov spaces, and the like.

The Fourier multipliers are not dealt with in this book. In order to emphasize the difference between them and the multipliers under consideration, we attach Sobolev's name to the latter. By coining the term Sobolev multipliers we just hint at various spaces of differentiable functions of Sobolev's type, being fully aware that Sobolev never worked on multipliers. After all, Fourier never did either. Sobolev multipliers arise in many problems of analysis and theories of partial differential and integral equations. Coefficients of differential operators can be naturally considered as multipliers. The same is true for symbols of more general pseudo-differential operators. Multipliers also appear in the theory of differentiable mappings preserving Sobolev spaces. Solutions of boundary value problems can be sought in classes of multipliers. Because of their algebraic properties, multipliers are suitable objects for generalizations of the basic facts of calculus (theorems on implicit functions, traces and extensions, point mappings and their compositions etc.) Moreover, some basic operators of harmonic analysis, like the classical maximal and singular integral operators, act in certain classes of multipliers. We believe that the calculus of Sobolev multipliers provides an adequate language for future work in the theory of linear and nonlinear differential and pseudodifferential equations under minimal restrictions on the coefficients, domains, and other data. Before the 1970s, the word multiplier was usually associated with the name of Fourier, and a deep theory of  $L_p$ -Fourier multipliers created by Marcinkiewicz, Mihlin, Hörmander et al was quite popular. As for the multipliers preserving a space of differentiable functions, only a few isolated results were known (Dezinatz and Hirschman [DH], Hirschman [Hi1], [Hi2], Strichartz [Str], Polking [Poll], Peetre [Pe2]), while the multipliers in pairs of such spaces were not considered at all.

The first (and the only one for the time being) attempt to work out a more or less comprehensive theory of multipliers acting either in one or in a pair of spaces of Sobolev type was undertaken by the authors in the late 1970s and early 1980s [Maz10], [Maz12], [MSh1]-[MSh16]. Results of that theory were collected in our monograph "Theory of Multipliers in Spaces of Differentiable Functions" (Pitman, 1985) [MSh16]. During the last two decades, we continued to work in the area, adding new results and developing further applications [Sh2]-[Sh14], [MSh17]-[MSh23]. We wish to reflect the present state of our theory in this book. An essential part of the aforementioned monograph is also included here. No results concerning multipliers in spaces of analytic functions are mentioned in what follows, in contrast to [MSh16]. To describe progress in this area achieved during the last twenty five years would require a disproportionate growth of the book. Structure of the book. The book consists of two parts. Part I is devoted to the theory of multipliers and covers the following topics: Trace inequalities Analytic characterization of multipliers Relations between spaces of Sobolev multipliers and other function spaces Maximal subalgebras of multiplier spaces Traces and extensions of multipliers Essential norm and compactness of multipliers Miscellaneous properties of multipliers (spectrum, composition and implicit function theorems, point mappings preserving Sobolev spaces, etc.) In Part II we dwell upon several applications of this theory. Their list is as follows:

Continuity and compactness of differential operators in pairs of Sobolev spaces Multipliers as solutions to linear and quasilinear elliptic equations Higher regularity in the single and double layer potential theory for Lipschitz domains B, regularity of the boundary in  $L_p$ -theory of elliptic boundary value problems Singular integral operators in Sobolev spaces Each chapter starts with a short introductory outline of the included material. Readership. The volume is addressed to mathematicians working in functional analysis

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and in the theories of partial differential , integral , and pseudo-differential operators. Prerequisites for reading this book are undergraduate courses in these subjects. Acknowledgments. V. Maz'ya was partially supported by the National Science Foundation ( Grant DMS-0500029 , USA ) and EPSRC ( Grant EP/F005563/1 , UK ) . T. Shaposhnikova gratefully acknowledges support from the Swedish Natural Science Research Council ( VR ) .

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### 内容概要

《索伯列夫乘子理论》旨在为读者全面讲述微分函数空间对点乘子理论。这个理论是在过去的三十年中通过众多学者大量积累发展起来的，《索伯列夫乘子理论》是前人结果的延伸和扩展。

第一部分介绍了乘子理论，囊括了众多理论和概念，如，迹不等式、乘子的解析特性、索伯列夫乘子空间和其他空间之间的关系、乘子空间最大子代数、迹和乘子扩展、乘子的范数和紧性以及乘子的综合特性；第二部分包括了该理论的大量应用，索伯列夫空间对微分算子的连续性和紧性；乘子作为线性和伪线性双曲方程的解；lipschitz域中单层和双层势能理论的高级正则性和双曲边界值问题 $L_p$ 理论中边界正则性；索伯列夫空间中的奇异积分算子。

这部著作综合性强，文笔流畅，结构紧凑，是泛函分析，偏微分方程和伪微分算子等相关数学专业不可多得的教材和参考书。

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### 作者简介

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他是托福考生必备参考资料——《新托福黄金精选阅读》的整理者，也是《新托福真题详解系列》以及《去美国读本科》的书籍主编。

常以老派的知识分子自居，高度近视，精力充沛，热爱红牛。

自诩为浑身赘肉而有问必答的热心人士。

大学时代他就开始了兼职英语老师的职业生涯，讲授包括托福在内的诸多出国留学考试辅导课程。

他讲课激情四射、风趣幽默，在强调英语综合能力的同时，擅长引导学生找到属于自己的解题技巧与思考方式，并将这些方法升华且融化到实际的做题之中去，已成功帮助上万名学生实现了解题思路“从无到有，从有到无”的质的飞跃。

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