<<几何测度论>>

图书基本信息

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前言

Singular geometry governs the physical universe： soap bubble clusters meeting along singular curves， black holes， defects in materials， chaotic turbulence， crys- tal growth. The governing principle is often some kind of energy minimization. Geometric measure theory provides a general framework for understanding such minimal shapes， a priori allowing any imaginable singularity and then proving that only certain kinds of structures occur.

Jean Taylor used new tools of geometric measure theory to derive the singular structure of soap bubble clusters and sea creatures， recorded by J. Plateau over a century ago （see Section 13.9）. R. Schoen and S.-T. Yau used minimal surfaces in their original proof of the positive mass conjecture in cosmology， recently extended to a proof of the Riemannian Penrose Conjecture by H. Bray. David Hoffman and his collaborators used modern computer technology to discover some of the first new complete embedded minimal surfaces in a hundred years （Figure 6.1.3）， some of which look just like certain polymers. Other mathematicians are now investigating singular dynamics， such as crystal growth. New software computes crystals growing amidst swirling fluids and temperatures， as well as bubbles in equilibrium， as on the front cover of this book. （See Section 16.8.） In 2000， Hutchings， Morgan， Ritorr， and Ros announced a proof of the Double Bubble Conjecture， which says that the familiar double soap bubble provides the least-area way to enclose and separate two given volumes of air. The planar case was proved by my 1990 Williams College NSF "SMALL" undergraduate research Geometry Group [Foisy et al.]. The case of equal volumes in R3 was proved by Hass， Hutchings， and Schlafly with the help of computers in 1995. The general R3 proof has now been generalized to Rn by Reichardt. There are partial results in spheres， tori， and Gauss space， an important example of a manifold with density （see Chapters 18 and 19）. This little book provides the newcomer or graduate student withan illustrated introduction to geometric measure theory： the basic ideas， terminology， and results. It developed from my one-semester course at MIT for graduate students with a semester of graduate real analysis behind them. I have included a few fundamental arguments and a superficial discussion of the regularity theory， but my goal is merely to introduce the subject and make the standard text. Geometric Measure Theory by H. Federer. more accessible.

内容概要

Singular geometry governs the physical universe： soap bubble clusters meeting along singular curves， black holes， defects in materials， chaotic turbulence， crys- tal growth. The governing principle is often some kind of energy minimization. Geometric measure theory provides a general framework for understanding such minimal shapes， a priori allowing any imaginable singularity and then proving that only certain kinds of structures occur.

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章节摘录

14.20 Theorem （Hutchings et al. Theorem 7.1） The standard double bubble in R3is the unique area-minimizing double bubble for prescribed volumes. Proof Let B be an area-minimizing double bubble. By Corollary 14.11 and Propo- sition 14.19, either both regions are connected or one of larger volume and smaller pressure is connected and the other of smaller volume and larger pressure has two components. By the Hutchings structure theorem, 14.10, B is either as in Figure 14.16.1 or as in Figure 14.17.1. By 14.16 and 14.18, B must be the standard double bubble. Remark Although the final competitors are proved unstable, earlier steps such as symmetry （14.3） assume area minimization. It remains conjectural whether the standard double bubble is the unique stable double bubble. 14.21 Open Questions It is conjectured by Hutchings et al. that the stan- dard double bubble in Rn is the unique stable double bubble. Sullivan [Sullivan and Morgan, Proposition 2] has conjectured that the standard k-bubble in Rn （k < n + 1） is the unique minimizer enclosing k regions of prescribed volume. For now, even the standard triple bubble in R3 （Figure 13.3.1） seems inaccessible. 14.22 Physical Stability As explained in Section 13.14, the technically correct physical soap cluster problem is to minimize the Helmholtz free energy F = U - TS to enclose and separate given quantities rather than volumes of gas （at fixed temperature T）, although the difference is negligible in practice. Here U is surface energy and S is entropy of the enclosed gas. To show that every round sphere minimizes F for a single given quantity of gas, since a round sphere minimizes surface area and hence U for fixed volume, it suffices to show that the number N of gas moles is an increasing function of volume v, which holds by scaling if for example N is proportional to Pun for n > 1/3 （for an ideal gas n = 1）. Here is the similar result for double bubbles in more detail: Proposition Assuming that the number N of gas moles is proportional to P for n > 1/3, a standard double bubble minimizes Helmholtz free energy for enclosing and separating two given masses of air. Proof The Helmholtz free energy of a double bubble depends on the surface area and the volumes. For given volumes, the standard double bubble minimizes surface energy （Theorem 14.20） and hence F. As either volume goes to 0, the entropy S goes to minus infinity and F goes to infinity. As either volume goes to infinity, the surface area and hence F go to infinity.